Deembedding of filters in multiplexers via rational approximation and interpolation

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***Abstract*—In this paper we present a method to recover electrical parameters of filters embedded in a multiplexer for which scattering measurements are given. Unlike other approaches proposed for this problem, this method does not require a priori knowledge of the scattering parameters of the junction. This feature renders the procedure well suited for tuning purposes or for fault diagnosis. Technically the algorithm starts with a rational approximation step, in order to derive a rational representation of certain scattering parameters of the multiplexer. This representation is then used in a second step to identify the electrical model of each filter. This second step relies on a rational interpolation technique used to extract the filter's responses.**

***Index Terms*—Diplexer, filter tuning, fault diagnosis, Deembedding, Rational Approximation, Padé interpolation.**

I. Introduction

Microwave multiplexers are present in nearly every transmission or reception unit of communication systems. The complex reciprocal loading effects of the filters connected via the junction (see Fig. 1) make their synthesis a difficult task: the latter commonly relies on computer-driven simulation, in the circuital and full wave domain, that are coupled to optimization methods [1]. The practical realization of such devices remains however a delicate matter, because of the inevitable dimension mismatch between the synthesized multiplexer and the realized one. The filters are therefore equipped with tuning elements (e.g. screws, irises) that need to be adjusted in the final manufacturing phase. When filters can be accessed and measured at their two ports, methods based on rational approximation have been developed ([2], [3]) to extract the electrical parameters of the measured filter, and have led to efficient tuning procedures. By contrast, when tuning a multiplexer, detaching filters from the common junction is however hardly a possible or suitable option. Note also that in the early stages of a fault diagnosis procedure on a damaged device, the ability to identify precisely the malfunctioning filter(s), without disassembling the entire multiplexer, might also be of great value. For all these reasons, important efforts have been spent on tuning techniques for multiplexers which rely only on external scattering measurements. Most of them are based on neural networks or on the minimization of a tuning criterion ([4], [5]), which however might suffer from the presence of local minima and usually give no real insight about the internal state of the hardware. Other approaches ([6]) have therefore been proposed in order to derive the filter's electrical parameters: these however necessitate the additional knowledge of the junction's scattering parameters, that might not be at hand in practice.

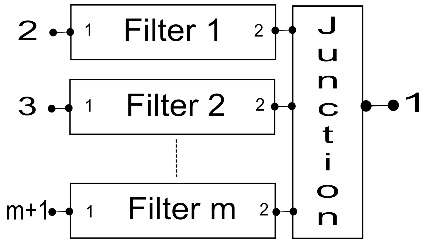

Fig. 1. Structure of the multiplexer and ports/filters numbering

We describe here a method requiring the sole knowledge of the multiplexer's external scattering measurements, the filter's order and the coupling geometries (in-line, box section, triplets etc...) they implement. The procedure's output are most of the electrical parameters of the filters up to the resonating frequency offsets of the resonators closest to the junction and their associated output couplings. It is also analytically proven that this is the best that can be done when starting from external measurements. From the latter our algorithm derives polynomial models of certain of the device's scattering parameters, from which the rational scattering matrices of the filters can be extracted. Eventually the filter's scattering matrices extracted in this way are synthesized in terms of electrical circuits of coupled resonators with the specified topologies, yielding the coupling matrix of each filter.

II. Filter's response extraction

We consider the situation depicted in Figure 2:

- a filter is loaded on its second port by a unknown load
- we have access to the reflexion parameter $S_{1,1}$ of this system (filter+load) at port one of the filter
- we know about the filter's order N, its coupling topology, and the location of its transmission zeros

Fig. 2. Filter with an arbitrary load on port 2

This section will try to answer the following natural question: under these circumstances, what can we say about the filter's response itself? We call $F_{1,1}, F_{1,2}, F_{2,2}$ the filter's scattering parameters, and $L_{1,1}$ the reflexion parameter of the load. With these notations the reflexion parameter of the overall system expresses as:

$$S_{1,1} = F_{1,1} + \frac{(F_{1,2})^2 L_{1,1}}{1 - L_{1,1}F_{2,2}} \tag{1}$$

Evaluating the preceding expression at a transmission zero ω_c of the filter is of interest in our situation, as it readily yields:

$$\begin{cases} F_{1,1}(\omega_c) = S_{1,1}(\omega_c) \\ F'_{1,1}(\omega_c) = S'_{1,1}(\omega_c) \end{cases} \tag{2}$$

This indicates that the values and the derivatives of the reflexion coefficient of the filter at one of its finite or infinite transmission zeros can be deciphered from those of the system's reflexion at the same point. Note that if ω_c is a zero of multiplicity m then the list of equalities (2) can be continued up to a derivative order $2m-1$. This remark will be useful for transmission zeros at infinity: for example a filter of order N with an in-line coupling topology has a zero of order N at infinity. The rationality of $F_{1,1,} = p/q$ leads us to consider the following interpolation problem: supposing $(\omega_1, \dots \omega_l)$ are all the filter's transmission zeros of respective order $(m_1, \dots m_l)$ ($\sum m_k = N$), find all the polynomials pairs (p, q) satisfying:

$$\forall k \in \{1 \dots l\} \; \forall i \in \{0 \dots 2m_k - 1\} \; \left(\frac{p}{q}\right)^{(i)} (\omega_k) = S_{1,1}^{(i)}(\omega_k) \tag{3}$$

The system (3) defines a rational interpolation problem, of Padé multipoint type [7], and can be solved classically using elementary methods from linear algebra. The system of equations (3) contains $2N$ equations compared to the $2N+2$ unknowns corresponding to the coefficients of polynomials p and q. It is therefore no surprise that the following holds:

Proposition 2.1: All the polynomial pairs (p, q) of degree at most N that solve (3) form a two dimensional space[1]. In particular there exist (p_1, q_1) and (p_2, q_2), such that for all complex numbers (α, β), the pair (p, q) defined as:

$$\begin{aligned} p &= \alpha p_1 + \beta p_2 \\ q &= \alpha q_1 + \beta q_2 \end{aligned} \tag{4}$$

is a solution to (3). Moreover any two pairs of distinct solutions satisfy,

$$p_1 q_2 - q_1 p_2 = \gamma^2 r^2 \tag{5}$$

where r is the transmission polynomial

$$r = \prod_{\omega_k \neq \infty} (s - \omega_k)^{m_k}$$

and γ a complex number (depending on the special choice of the solution pair).

[1]Strictly mathematically speaking this assertion is true, only generically (for almost all), with respect to the interpolation data $S_{1,1}^i(\omega_k)$

The lack of uniqueness of the identified filter's reflexion coefficient is coherent with the following remark: one can always intercalate between the filter and the load (see Fig. 2) an arbitrary constant (independent of the frequency) chain matrix followed by its inverse, and this without changing the reflexion parameter $S_{1,1}$ of the system. The filter's response is therefore, at most recoverable up to the chaining of a constant chain matrix at its port two. The following proposition shows that this is the sole uncertainty on the filter's response:

Proposition 2.2: In the notations of proposition (2.1), and for any two pairs $a = (p_1, q_1)$ and $b = (p_2, q_2)$ solutions of (3), the scattering matrix:

$$F_{a,b} = \frac{1}{q_1} \begin{pmatrix} p_1 & \gamma r \\ \gamma r & q_2 \end{pmatrix} \tag{6}$$

is a possible solution to our de-embedding problem, in the sense that:

- $F_{a,b}$ has ω_k as transmission zeros
- $F_{a,b}$ is of McMillan degree at most N (its determinant is a rational function of degree at most N)
- It satisfies equation (2) at every transmission zero ω_k

Moreover every solution in this sense to the de-embedding problem can be obtained like that. Eventually, for any two choices of solutions (a, b) and (a', b'), $F_{a,b}$ and $F_{a',b'}$ differ only up to the chaining of a constant chain matrix on their second port.

In order to access the electrical parameters of the filter we need now to consider circuital realizations of the derived rational scattering matrices. The considered circuital realization consists in the classical low-pass prototype [8], made of coupled resonators with a specified coupling topology. Following proposition indicates that most of the electrical parameters of the filter are recovered.

Proposition 2.3: Suppose that the filter has at most $N-2$ transmission zeros at finite frequencies, in order to admit a circuital realization with no source-load coupling [8]. Suppose that the scattering matrix F is a possible solution of the de-embedding problem (in the sense of proposition 2.2) and admits a circuital representation characterized by a $(N+2)\times(N+2)$ coupling matrix M. Then any other scattering matrix F', also solution of the de-embedding problem, admits a circuital representation with coupling matrix M' that differs from M only for:

- the frequency offset of the last cavity of the filter (nearest to the load), that is the value $M_{N,N}$ of the coupling matrix
- the value of the output coupling represented by $M_{N,L}$ in the coupling matrix (see [8])

The preceding proposition indicates that the uncertainty on the scattering matrix of the filter has a very localized impact on its circuital representation. This is coherent with the fact that this uncertainty weighs on the value of a constant chain matrix plugged at its output port.

III. De-embedding filters from a multiplexer

The problem of de-embedding filters from a multiplexer, when starting from external measurements, is in many aspects similar to the simplified situation considered in section II. If we call G the scattering matrix of the multiplexer:

- Each filter appears to be loaded at its second port by a component regrouping the remaining filters connected via the junction
- For all these sub-systems, composed of a filter loaded by an unknown component, the reflexion at port 1 is known. For the first filter, it is for example given by the scattering parameter $G_{2,2}$, for the second filter by $G_{3,3}$ etc...
- The transmission zeros, for example of filter 1, are also present in the transmission parameters $G_{2,1}, G_{2,3}, G_{2,4} \dots$. This property, as we will see, can be used to locate them.

The core idea of the de-embedding algorithm is to perform a line-wise rational approximation of the scattering measurement of G: we will detail it from the perspective of the de-embedding problem of filter 1.

We start with a rational approximation of the line $[G_{2,2}, G_{2,1}, G_{2,3} \dots]$. The order of the rational approximation is chosen so as to get a proper fitting of the measurements and is in general higher than the filter's order: an additional increase in degree is introduced to approximate the effects of the loading. If the filter is expected to have finite transmission zeros, they can be identified using the following remark: the finite transmission zeros of filter 1 are the common zeros of the transmission parameters $G_{2,1}, G_{2,3}, \dots$ (apart from unlikely situations where all the filters have the same transmission zeros, or the junction itself has a transmission zero). Their locations can therefore be obtained by analyzing the common zeros of the numerators of the rational approximations of $G_{2,1}, G_{2,3}, \dots$. Eventually the values and derivatives of the right hand term of (3) will be provided by evaluating the rational approximation of $G_{2,2}$. Extraction of coupling parameters is then obtained following the procedure outlined in section II.

IV. Practical example: measured diplexer

We consider an example, made of a manifold diplexer manufactured in one piece shown in Figure.3.

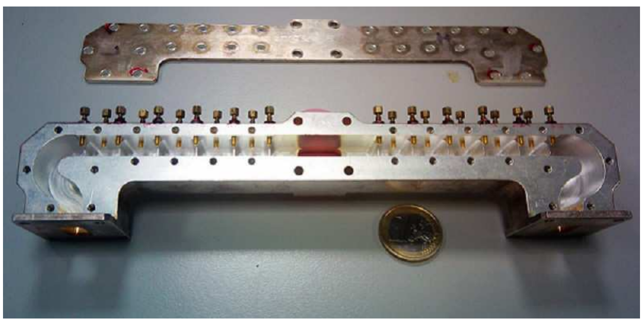

Fig. 3. Manufactured diplexer. Tuning screws are used to adjust couplings and resonating frequencies of the cavities

Both filters are of order 5 and implement an in-line coupling topology: all transmission zeros are therefore at infinity. To test our approach we ran several measurements of all the device's scattering parameters, while screwing in and out some tuning screws. Tuning screws entering laterally into the coupling windows between two cavities act like small capacities: when turned in, the corresponding coupling tend to augment. One can also expect the resonating cavities of the adjacent cavities to be slightly affected by the coupling screws. In order to get a reference, we therefore ran a first identification of filter 1 and found the following coupling matrix:

$$\begin{matrix} 0 & +1.015 & 0 & 0 & 0 & 0 & 0 \\ +1.015 & -0.006 & +0.839 & 0 & 0 & 0 & 0 \\ 0 & +0.839 & +0.012 & +0.631 & 0 & 0 & 0 \\ 0 & 0 & +0.631 & +0.032 & +0.617 & 0 & 0 \\ 0 & 0 & 0 & +0.617 & +0.145 & +0.860 & 0 \\ 0 & 0 & 0 & 0 & +0.860 & -0.136 & +1.091 \\ 0 & 0 & 0 & 0 & 0 & +1.091 & 0 \end{matrix}$$

We then turned in the coupling screw between the 3^{rd} and the 4^{th} cavity, measured the diplexer, and identified the coupling matrix of the first filter as:

$$\begin{matrix} 0 & +1.014 & 0 & 0 & 0 & 0 & 0 \\ +1.014 & -0.006 & +0.839 & 0 & 0 & 0 & 0 \\ 0 & +0.839 & +0.012 & +0.628 & 0 & 0 & 0 \\ 0 & 0 & +0.628 & -0.023 & +0.692 & 0 & 0 \\ 0 & 0 & 0 & +0.692 & +0.080 & +0.864 & 0 \\ 0 & 0 & 0 & 0 & +0.864 & -0.151 & +0.960 \\ 0 & 0 & 0 & 0 & 0 & +0.960 & 0 \end{matrix}$$

The predicted increase of the coupling $M_{3,4}$ is confirmed by the identification as well as a variation of the resonating frequency of the 4^{th} cavity. As explained in section II the identified element $M_{5,5}$ and $M_{N,L}$ should not be given attention to. Other similar examples, handling for example triplexer filters implementing finite transmission zeros in a "quartet" topology", have been de-embedded with success but are not reported here due to lack of space.

V. Conclusions

In this paper we presented a method to extract electrical parameters of the multiplexer's filters through the sole knowledge of external scattering measurements. The method can be seen as an analogue, in the context of system identification, of Darlington's extraction procedure [9] used in the context of filter synthesis. The method was validated on practical examples and is suited for applications in the context of computer assisted tuning and fault diagnosis. Although it has not been tested yet, application of the methodology to other complex devices combining filters that can be accessed by only one of their port seems promising.

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